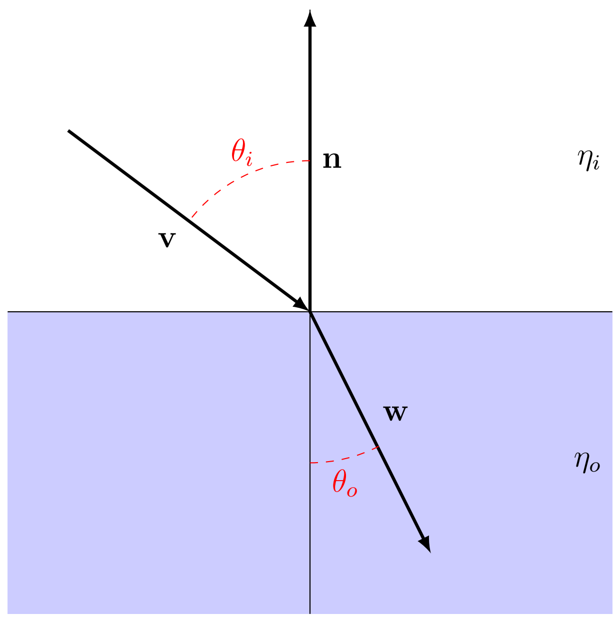
CS130 - Reflections and transparency

Name: ______________________________ SID: __________________

1. Given the vector $\mathbf{u} = \langle 1, 4 \rangle$ and the unit vector $\mathbf{n} = \langle -\frac{3}{5}, \frac{4}{5} \rangle$, decompose $\mathbf{u}$ into component $\mathbf{u}_\perp$ perpendicular to $\mathbf{n}$ and component $\mathbf{u}_\parallel$ parallel to $\mathbf{n}$ such that $\mathbf{u} = \mathbf{u}_\perp + \mathbf{u}_\parallel$.



In the figure above, a ray originally in the air (index of refraction η_i) enters a transparent material (index of refraction η_o). The ray enters along direction $\mathbf{v}$ and leaves along direction $\mathbf{w}$. You are given that $\|\mathbf{v}\| = 1$ and $\|\mathbf{n}\| = 1$. You will construct $\mathbf{w}$ such that $\|\mathbf{w}\| = 1$. $\mathbf{w}$ lies in the same plane as $\mathbf{n}$ and $\mathbf{v}$.

2. Snell's law states that $\eta_i \sin\theta_i = \eta_o \sin\theta_o$. Express this equation in terms of the vectors $\mathbf{v}$, $\mathbf{n}$, and $\mathbf{w}$ using cross products (no dot products).

3. Taking advantage of the fact that $\mathbf{w}$, $\mathbf{n}$, and $\mathbf{v}$ lie in the same plane, we can write $\mathbf{w} = a\mathbf{v} + b\mathbf{n}$. Using your result from the previous problem, solve for a. Note that you will only be able to solve for a up to a sign.

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4. Let $\mathbf{t}$ be a vector orthogonal to $\mathbf{n}$ as shown in the figure. Taking the dot product of $\mathbf{w} = a\mathbf{v} + b\mathbf{n}$ by $\mathbf{t}$, deduce the sign of a.

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5. Using $\|\mathbf{w}\|^2 = 1$ to derive a quadratic equation in b. Solve this for b, which should give you two solutions. We will select the solution we want later.

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6. If $\mathbf{v} = -\mathbf{n}$, then we should get $\mathbf{w} = \mathbf{v}$ as our solution. Use this special case to deduce the correct sign for b. Using the a and b you derived, write out $\mathbf{w}$.

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7. Based on your formula for $\mathbf{w}$, deduce the conditions under which complete internal reflection occurs.

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8. What happens as the index of refraction of the sphere in `08.txt` is made closer to the index of refraction of the air? Support your conclusion by showing a sequence of renders. What happens when they are equal?

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